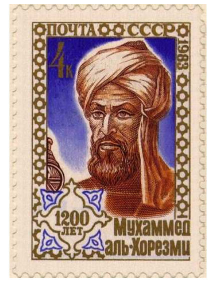
Traduire et interpréter ce texte avec les connaissances du collège

Having introduced the natural numbers, al-Khwarizmi introduces the main topic of this first section of his book, namely the solution of equations. His equations are linear or quadratic and are composed of units, roots and squares. For example, to al-Khwarizmi a unit was a number, a root was x, and a square was x^2. However, although we shall use the now familiar algebraic notation in this article to help the reader understand the notions, Al-Khwarizmi's mathematics is done entirely in words with no symbols being used.

He first reduces an equation (linear or quadratic) to one of six standard forms :

- Squares equal to roots ?
- Squares equal to numbers ?
- Roots equal to numbers ?
- Squares and roots equal to numbers ; e.g. $x^2+10x = 39$?
- Squares and numbers equal to roots ; e.g. $x^2+21 = 10x$?
- Roots and numbers equal to squares ; e.g. $3x+4 = x^2$?



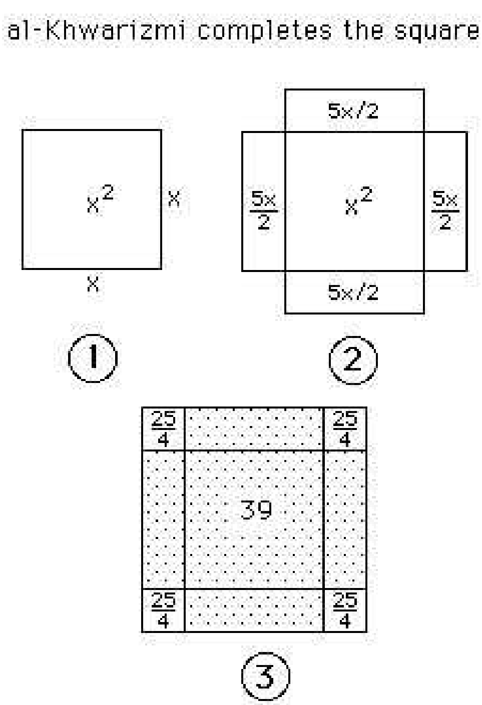
The reduction is carried out using the two operations of al-jabr and al-muqabala. Here "al-jabr" means "completion" and is the process of removing negative terms from an equation. For example, using one of al-Khwarizmi's own examples, "al-jabr" transforms

$$x^2 = 40x - 4x^2 \qquad \text{into} \qquad 5x^2 = 40x$$

The term "al-muqabala" means "balancing" and is the process of reducing positive terms of the same power when they occur on both sides of an equation. For example, two applications of "al-muqabala" reduces

$$50+3x+x^2 = 29+10x \qquad \text{to} \qquad 21+x^2 = 7x$$

(one application to deal with the numbers and a second to deal with the roots). Al-Khwarizmi then shows how to solve the six standard types of equations. He uses both algebraic methods of solution and geometric methods. For example to solve the equation $x^2+10x = 39$ he writes... a square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows : what is the square which combined with ten of its roots will give a sum total of 39 ?



The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64.
Having taken then the square root of this which is 8, subtract from it half the roots, 5 leaving 3. The number three therefore represents one root of this square, which itself, of course is 9. Nine therefore gives the square.
The geometric proof by completing the square follows. Al-Khwarizmi starts with a square of side x, which therefore represents x^2 (Figure 1). To the square we must add $10x$ and this is done by adding four rectangles each of breadth $\frac{10}{4}$ and length x to the square (Figure 2).
Figure 2 has area $x2+10x$ which is equal to 39.
We now complete the square by adding the four little squares each of area $\frac{5}{2}\times\frac{5}{2} = \frac{25}{4}$.
Hence the outside square in Fig 3 has area $4\times\frac{25}{4}+39 = 25+39 = 64$.
The side of the square is therefore 8.
But the side is of length $\frac{5}{2}+x+\frac{5}{2}$ so $x+5 = 8$, giving $x = 3$.

Résoudre de la même façon l'équation : $x^2+12x = 133$ puis $x^2+3x = \frac{133}{4}$